

9.3

Solving Multi-Step Inequalities



Focus on...

After this lesson, you will be able to...

- solve multi-step linear inequalities and verify their solutions
- compare the processes for solving linear equations and linear inequalities
- solve problems involving multi-step linear inequalities

Bryan's grandmother gave him \$60 to spend at the go-cart track. Each lap at the track costs \$3.50. How many laps can he buy if he wants to have at least \$20 left over to buy lunch for himself and his grandmother?

Describe different strategies you could use to solve this problem.

Explore Multi-Step Inequalities

1. **a)** Estimate the number of laps Bryan can buy.
b) How can your strategy help you set up and solve an inequality?
2. Develop an inequality that can be used to determine the number of laps Bryan can buy.
3. **a)** Outline a strategy for solving your inequality.
b) Use your strategy to determine the solution. Show your steps.
c) How can you use the solution to solve the problem?

Reflect and Check

4. **a)** Is the solution to the problem a single value? Or is it a set of several possible values? Explain.
b) What words in the problem indicate that you could model it using an inequality? Explain.
5. **a)** Compare the strategy you used to solve the multi-step inequalities with that of a classmate. Which strategy do you prefer? Explain.
b) How did your knowledge of solving linear equations help you solve these inequalities?

Link the Ideas

Example 1: Solve Multi-Step Inequalities

- a) Solve $\frac{x}{4} + 3 > 8$. Show your solution algebraically and graphically.

Verify the solution.

- b) Solve $-3x - 10 \leq 5x + 38$, and verify the solution.
c) Solve $-2(x + 3) \leq 10x + 18$, and verify the solution.

Solution

- a) Use the same process to solve a multi-step inequality as for solving a linear equation.

$$\frac{x}{4} + 3 > 8$$

$$\frac{x}{4} + 3 - 3 > 8 - 3$$

$$\frac{x}{4} > 5$$

$$\frac{x}{4} \times 4 > 5 \times 4$$

$$x > 20$$

The number line shows the solution.



Verify the solution:

Substitute the boundary point 20 to check that both sides are equal.

$$\frac{x}{4} + 3 = 8$$

$$\frac{20}{4} + 3 = 8$$

$$5 + 3 = 8$$

$$8 = 8$$

Substitute a value greater than 20. If a true statement results, then the inequality symbol is correct.

$$\frac{x}{4} + 3 > 8$$

$$\frac{24}{4} + 3 > 8$$

$$6 + 3 > 8$$

$$9 > 8$$

Since both statements are true, the solution $x > 20$ is correct.

What steps would you follow to isolate the variable?

You can isolate the variable on the left or the right side of the inequality. Which strategy do you prefer?

b) Isolate the variable.

Isolate the Variable on the Left Side

$$\begin{aligned}
 -3x - 10 &\leq 5x + 38 \\
 -3x - 10 + 10 &\leq 5x + 38 + 10 \\
 -3x &\leq 5x + 48 \\
 -3x - 5x &\leq 5x + 48 - 5x \\
 -8x &\leq 48 \\
 \frac{-8x}{-8} &\geq \frac{48}{-8} \\
 x &\geq -6
 \end{aligned}$$

Isolate the Variable on the Right Side

$$\begin{aligned}
 -3x + 3x - 10 &\leq 5x + 38 + 3x \\
 -10 &\leq 8x + 38 \\
 -10 - 38 &\leq 8x + 38 - 38 \\
 -48 &\leq 8x \\
 \frac{-48}{8} &\leq \frac{8x}{8} \\
 -6 &\leq x
 \end{aligned}$$

Verify the solution:

Substitute the boundary point -6 to check that both sides are equal.

$$\begin{aligned}
 -3x - 10 &= 5x + 38 \\
 -3(-6) - 10 &= 5(-6) + 38 \\
 18 - 10 &= -30 + 38 \\
 8 &= 8
 \end{aligned}$$

Substitute a value greater than -6 . If a true statement results, then the inequality symbol is correct.

$$\begin{aligned}
 -3x - 10 &\leq 5x + 38 \\
 -3(0) - 10 &\leq 5(0) + 38 \\
 0 - 10 &\leq 0 + 38 \\
 -10 &\leq 38
 \end{aligned}$$

Since both statements are true, the solution $x \geq -6$ is correct.

c) *Method 1: Use the Distributive Property*

$$\begin{aligned}
 -2(x + 3) &\leq 10x + 18 \\
 -2x - 6 &\leq 10x + 18 \\
 -2x - 6 - 10x &\leq 10x + 18 - 10x \\
 -12x - 6 &\leq 18 \\
 -12x - 6 + 6 &\leq 18 + 6 \\
 -12x &\leq 24 \\
 \frac{-12x}{-12} &\geq \frac{24}{-12} \\
 x &\geq -2
 \end{aligned}$$

Method 2: Divide First

You can divide first by -2 .

$$\begin{aligned}
 -2(x + 3) &\leq 10x + 18 \\
 \frac{-2(x + 3)}{-2} &\geq \frac{10x + 18}{-2} \\
 x + 3 &\geq -5x - 9 \\
 x + 5x + 3 &\geq -5x - 9 + 5x \\
 6x + 3 &\geq -9 \\
 6x + 3 - 3 &\geq -9 - 3 \\
 6x &\geq -12 \\
 \frac{6x}{6} &\geq \frac{-12}{6} \\
 x &\geq -2
 \end{aligned}$$

Verify the solution:

Substitute the boundary point -2 .

$$\begin{aligned} -2(x + 3) &= 10x + 18 \\ -2((-2) + 3) &= 10(-2) + 18 \\ -2(1) &= -20 + 18 \\ -2 &= -2 \end{aligned}$$

Substitute a value greater than -2 , such as 0 .

$$\begin{aligned} -2(x + 3) &\leq 10x + 18 \\ -2(0 + 3) &\leq 10(0) + 18 \\ -2(3) &\leq 0 + 18 \\ -6 &\leq 18 \end{aligned}$$

Choose numbers you find easy to work with.

Since both statements are true, the solution $x \geq -2$ is correct.

Show You Know

Solve each inequality and verify the solution.

- a) $4x + 11 > 35$ b) $5 - 2x > 10x + 29$ c) $4(x - 2) \geq 5x - 12$

WWW Web Link

To practise solving inequalities using multiplication and division, go to www.mathlinks9.ca and follow the links.

Example 2: Solve a Problem Using Inequalities

Sarah has offers for a position as a salesperson at two local electronics stores. Store A will pay a flat rate of \$55 per day plus 3% of sales. Store B will pay a flat rate of \$40 per day plus 5% of sales. What do Sarah's sales need to be for store B to be the better offer?

- a) Write an inequality to model the problem.
b) Solve the inequality and interpret the solution.



Did You Know?

Sales people often work on *commission*, which is a form of payment based on the amount of their sales.

Solution

- a) Let s represent the value of Sarah's sales for a particular day. Determine s when the pay for store B is greater than the pay for store A.

$$\begin{aligned} \text{Pay for store B} &> \text{Pay for store A} \\ 40 + 5\% \text{ of sales} &> 55 + 3\% \text{ of sales} \\ 40 + 0.05s &> 55 + 0.03s \end{aligned}$$

What do 0.03 and 0.05 represent in the inequality?

Strategies

What other strategy could you use to solve the problem?

$$\begin{aligned} \text{b) } & 40 + 0.05s > 55 + 0.03s \\ & 40 + 0.05s - 40 > 55 + 0.03s - 40 \\ & 0.05s > 15 + 0.03s \\ & 0.05s - 0.03s > 15 + 0.03s - 0.03s \\ & 0.02s > 15 \\ & \frac{0.02s}{0.02} > \frac{15}{0.02} \\ & s > 750 \end{aligned}$$

Sarah's pay will be higher at store B when her sales are greater than \$750. If she thinks that her sales will be more than \$750 on most days, then store B is the better offer.

Show You Know

Danny started his own computer repair business. He offers his customers two payment options. Option A has a base fee of \$40 plus \$8 per hour. Option B has no base fee but costs \$15 per hour. How many hours does a repair job have to take in order for option B to be less expensive?

- Model the problem using an inequality.
- After how many hours will option B be less expensive?



Key Ideas

- To solve a multi-step inequality, isolate the variable.

$$\begin{aligned} -3(x - 5) &\leq 3x + 9 \\ -3x + 15 &\leq 3x + 9 \\ -3x + 15 - 3x &\leq 3x + 9 - 3x \\ -6x + 15 &\leq 9 \\ -6x + 15 - 15 &\leq 9 - 15 \\ -6x &\leq -6 \\ \frac{-6x}{-6} &\geq \frac{-6}{-6} \\ x &\geq 1 \end{aligned}$$

Remember to reverse the inequality symbol, when multiplying or dividing by a negative number.

- Problems that involve comparing different options can often be modelled and solved using inequalities.

Check Your Understanding

Communicate the Ideas

1. Describe the similarities and differences between the process for solving a multi-step linear equation and a multi-step linear inequality. Discuss your answer with a classmate.
2. Consider the inequality $3x + 10 > 5x + 22$. Lindsay started to solve the inequality by subtracting $5x$ from both sides. Victoria told her to start by subtracting $3x$ from both sides.
 - a) Use Lindsay's approach to solve the inequality.
 - b) Use Victoria's approach to solve the inequality.
 - c) Are the solutions the same? Explain.
 - d) Explain why you think Victoria gave her advice. Is her reasoning helpful in solving the inequality? Explain.
 - e) Which method of solving the inequality do you prefer? Explain why.

Practise

For help with #3 to #7, refer to Example 1 on pages 361–363.

3. Solve each inequality and verify the solution.

- a) $5x - 19 < 36$
- b) $27 + 2x > -13$
- c) $3 \leq \frac{x}{5} - 7$

4. Determine the solution of each inequality.

- a) $-5y + 92 \geq 40$
- b) $2.2 > 10.6 + 4y$
- c) $\frac{y}{-6} - 2 < 16$
- d) $\frac{3}{2}x + 6 \leq 10\frac{4}{5}$

5. a) Verify that $x \geq 8$ is the correct solution to the inequality $3x + 11 \geq 35$.

- b) Verify that $x < -3$ is the correct solution to the inequality $24 - 5x > 39$.

6. Solve each inequality and verify the solution.

- a) $7x < 2x + 30$
- b) $10x - 22 \geq 8x$
- c) $-12x + 10 > 19 - 4x$
- d) $\frac{1}{2}(x + 5) > 22$

7. Determine each solution.

- a) $-2y > 8y - 20$
- b) $9y - 17 \leq 8 + 6.5y$
- c) $3.4 - 1.3y < 0.5y - 2.2$
- d) $\frac{3}{4}y - 1 \geq -\frac{1}{4}(1 - 2y)$

For help with #8 and #9, refer to Example 2 on pages 363–364.

8. For each situation

- choose a variable and explain what it represents
 - write an inequality
- a) A basketball team wants to buy new jerseys. Uniforms R Us charges \$50 per jersey. Jerseys Unlimited charges \$40 per jersey plus \$80 for a logo design. How many jerseys does the team need to buy for Jerseys Unlimited to be the better option?
 - b) Ann uses her cell phone to send text messages. The monthly charge for text messaging is currently \$15 plus \$0.05 per message sent. The company is offering a new plan that costs a flat rate of \$0.12 per text message. How many text messages does Ann need to send in order for the new plan to be the better option?

9. John is considering two paper delivery jobs. The *Advance* will pay \$10 plus \$0.05 for each paper delivered daily, and the *Times* will pay \$15 plus \$0.04 for each paper delivered daily. How many papers delivered each day would make the *Advance* the better offer?



- Write an inequality to model the problem.
- Solve the inequality and interpret the solution.

WWW Web Link

To learn how to solve inequalities using a graphing calculator, go to www.mathlinks9.ca and follow the links.

Apply

10. Kim is comparing the rates at two car rental companies for a one-day rental. She wants to determine how many kilometres she would need to drive for ABC Rentals to be the better rental option.

ABC Rentals

\$25 per day plus
\$0.14 per kilometre

It's a Deal Rentals

\$55 per day

- Estimate the number of kilometres that would make ABC Rentals the better option.
 - Represent the situation using an inequality.
 - Solve the inequality and interpret the solution.
 - Compare the solution with your estimate.
11. Kevin is comparing job offers at two stores. Dollar Deal offers \$8/h plus 10% commission. Great Discounts offers \$18/h with no commission. What do Kevin's weekly sales need to be in order for Dollar Deal to pay more? Assume that he works an 8-h day five days per week.

12. The student council is considering two different companies to print the school's yearbooks. Great Graphics charges \$250 plus \$12.25 per book. Print Express charges \$900 plus \$9.50 per book. How many orders for yearbooks would make Print Express the better option?
13. Greenway Golf Course offers two plans for paying for buckets of balls at the driving range. How many buckets of balls used per month make the members' plan the better deal?



14. Molly has a business making candles. Her business costs are \$200 plus \$0.70 per candle made. She charges her customers \$3.50 for each candle. If she sells all of the candles she makes, how many candles sold would allow her to make a profit?



15. Two full water storage tanks are being drained for maintenance. The first tank holds 800 L of water and drains at a rate of 18 L/min. The second tank holds 500 L of water and drains at a rate of 7 L/min. Use an inequality to determine when the first tank will contain less water than the second tank.
16. Rob and Ashley are riding their bicycles uphill. Currently, Rob is 5.7 km from the top and climbing at 0.24 km/min. Ashley is 4.5 km from the top and riding at 0.17 km/min.
- Estimate when Rob will be closer to the top than Ashley.
 - Use an inequality to determine when Rob will be closer to the top than Ashley.
- Extend**
17. Solve $\frac{2}{3}(2x - 5) < \frac{1}{2}(x + 2)$.
Show the solution on a number line.
18. If $2x + 5 > 10$ and $5x - 4 < 20$, determine the possible values of x . Show your solution on a number line.
19. Lauren charges \$12 to cut lawns for neighbours. It takes her 25 min to cut each lawn and 40 min per month to maintain her lawn mower. She wants to earn \$400 each month without working more than 16 h cutting lawns. How many lawns can Lauren cut in a month and stay within her guidelines? Use two inequalities to determine the range for the number of lawns that she can cut.
20. Ella's teacher asked which is greater, x or $-x$? Ella said that x is always greater than $-x$.
- Write an inequality to represent Ella's response and solve it. When, if ever, is Ella correct?
 - Ella's teacher explained that her response is correct for some values of x only. For what values of x is Ella incorrect? Give one specific solution where Ella is correct and one where she is incorrect.
21. Solve $-13 \leq 5 - 2x$ and $5 - 2x \leq 9$.
22. Given that $b < 0$, solve the inequality $3 > bx + 3$.

Math Link

An amusement park manager needs to ensure that the park is profitable. For the park to make a profit, the total revenue needs to be more than the total expenses.

There are fixed expenses and revenues that remain the same. There are also variable expenses and revenues that depend on the number of visitors.

The manager estimates operating expenses and revenues for the park per visitor. These are shown in the table. Assuming the park offers ten rides, fill in the missing information.

- What is the total of the variable expenses per visitor? What are the total fixed costs? Write an expression to represent the total expenses.
- What is the total of the variable revenues per visitor? What are the total fixed revenues? Write an expression to represent the total revenues.
- Develop and solve an inequality to determine the number of visitors needed per day to make a profit. Justify your solution mathematically.

Daily Expenses	
Total variable operating costs per visitor	\$15
Total fixed costs (\$5000 + \$1200 per ride)	
Daily Revenues	
Admission (includes ride pass) per visitor	\$38
Food per visitor	\$25
Souvenirs per visitor	\$10
Parking per visitor	\$10
Total variable revenues per visitor	
Fixed revenue from sponsorship	\$2500