9.1 Representing Inequalities

Focus on...

After this lesson, you will be able to...

- **represent single-variable linear inequalities verbally, algebraically, and graphically**
- **determine if a given number is a possible solution of a linear inequality**

Did You Know?

Zdeno Chara is the tallest person who has ever played in the NHL. He is 206 cm tall and is allowed to use a stick that is longer than the NHL's maximum allowable length.

The official rule book of the NHL states limits for the equipment players can use. One of the rules states that no hockey stick can exceed 160 cm.

What different ways can you use to represent the allowable lengths of hockey sticks?

Explore Inequalities

1. a) Show how you can use a number line to graph lengths of hockey sticks in centimetres. Use a convenient scale for the range of values you have chosen to show. Why did you select the scale you chose?

b) Mark the maximum allowable length of stick on your line.

2. a) Consider the NHL's rule about stick length. Identify three different allowable stick lengths that are whole numbers. Identify three that are not whole numbers. Mark each value on your number line.

b) Think about all the possible values for lengths of sticks that are allowable. Describe where all of these values are located on the number line. How could you mark all of these values on the number line?

3. a) Give three examples of stick lengths that are too long. Where are these values located on the number line?

b) Discuss with a partner how to state the possible length of the shortest illegal stick. Is it reasonable to have a minimum length for the shortest illegal stick? Why or why not?

Did You Know?

Most adult hockey sticks range from 142 cm to 157.5 cm in length.

Reflect and Check

4. The value of 160 cm could be called a boundary point for the allowable length of hockey sticks.

a) Look at the number line and explain what you think the term *boundary point* means.

b) In this situation, is the boundary point included as an allowable length of stick? Explain.

5. The allowable length of hockey sticks can be expressed mathematically as an **inequality**. Since sticks must be less than or equal to 160 cm in length, the linear inequality is $l \leq 160$, where l, in centimetres, represents the stick length.

Write an inequality to represent the lengths of illegal sticks. Discuss your answer with a classmate.

inequality

- a mathematical statement comparing expressions that may not be equal
- can be written using the symbol $<$, $>$, $\leq$, $\geq$, or $\neq$

Did You Know?

The world's largest hockey stick and puck are in Duncan, British Columbia. The stick is over 62 m in length and weighs almost 28 000 kg.

Link the Ideas

Reading an inequality depends on the inequality symbol used.

Inequality	Meaning
$a > b$	a is greater than b
$a < b$	a is less than b
$a \geq b$	a is greater than or equal to b
$a \leq b$	a is less than or equal to b
$a \neq b$	a is not equal to b

Literacy Link

Inequalities can be expressed three ways:

- *Verbally* using words. For example, "all numbers less than or equal to 0.75."
- *Graphically* using visuals, such as diagrams and graphs. For example,

- *Algebraically* using mathematical symbols. For example, $x \leq 0.75$.

Example 1: Represent Inequalities

Many jobs pay people a higher rate for working overtime. Reema earns overtime pay when she works more than 40 h a week.

a) Give four possible values that would result in overtime pay.

b) Verbally express the amount of time that qualifies for overtime as an inequality.

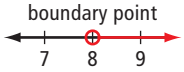
c) Express the inequality graphically.

d) Express the inequality algebraically.

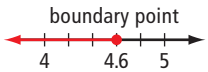
e) Represent the amount of time that does not qualify for overtime as an inequality. Express the inequality verbally, graphically, and algebraically.

boundary point

- separates the values less than from the values greater than a specified value
- may or may not be a possible value in a solution
- an open circle shows that the boundary point is not included in the solution



- a closed circle shows that the boundary point is included in the solution

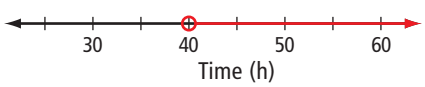


Solution

a) Reema does not qualify for overtime if she works exactly 40 h. She qualifies only if she works more than 40 h. Some examples include 40.5 h, 42 h, 46.25 h, and 50 h.

b) In order to qualify for overtime, Reema needs to work more than 40 h.

c) Draw a number line to represent the inequality graphically. Display the value 40 and values close to 40. The value 40 is a **boundary point**. This point separates the regular hours from the overtime hours on the number line. Draw an open circle at 40 to show the boundary point. Starting at 40, draw an arrow pointing to the right to show that the possible values of t are greater than but not equal to 40.



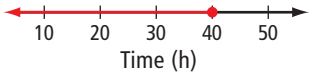
The open circle shows that the value 40 is not a possible value for the number of hours that qualify for overtime.

d) The inequality is $t > 40$, where t represents the amount of time, in hours, that Reema works in a week.

Which of the three representations of an inequality do you prefer?

e) Verbally: Reema does not qualify for overtime if the number of hours she works is less than or equal to 40 h.

Graphically: Draw a closed circle at 40. Draw an arrow pointing to the left of 40 to show the possible values of t less than or equal to 40.



The closed circle shows that 40 is a possible value for the number of hours that do not qualify for overtime.

Algebraically: Using t to represent the amount of time, in hours, that Reema works, $t \leq 40$.

Show You Know

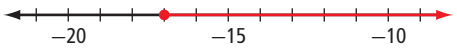
In many provinces, you must be at least 16 years of age to get a driver's licence.

a) Sketch a number line to represent the situation.

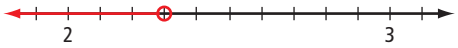
b) Represent the situation algebraically.

Example 2: Express Inequalities

a) Express the inequality shown on the number line verbally and algebraically.



b) Express the inequality shown on the number line algebraically.



c) Express the inequality $x \geq -\frac{4}{7}$ graphically.

d) Express the inequality $35 < n$ graphically.

Solution

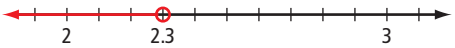
a) The number line shows a closed circle on -17 and an arrow to the right. This means values are the same as or larger than -17.

What does the arrow to the right represent? What does the closed circle represent?

Verbally: The number line indicates all the values greater than or equal to -17.

Algebraically: Using x as the variable, $x \geq -17$.

b) The space between 2 and 3 is divided into ten intervals, so each one represents 0.1 or $\frac{1}{10}$.



The number line shows an open circle on 2.3 and an arrow to the left. This indicates the values less than 2.3 but not including 2.3.

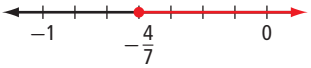
Using x as the variable, $x < 2.3$ or $x < 2\frac{3}{10}$ or $x < \frac{23}{10}$.

c) The inequality represents values greater than or equal to $-\frac{4}{7}$.

The boundary point is between -1 and 0.
Draw a number line with -1 and 0 labelled.
Divide the space between -1 and 0 into seven intervals.

Why do you divide into seven intervals?

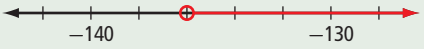
Draw a closed circle at $-\frac{4}{7}$. Draw an arrow to the right to indicate values that are greater than or equal to $-\frac{4}{7}$.



d) In this inequality, the variable is on the right. You can read the inequality as "35 is less than n." This is the same as saying n is larger than 35. Draw a number line showing an open circle on 35 and an arrow pointing to the right.

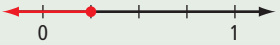
Show You Know

a) Express the inequality shown on the number line algebraically.



b) Represent the inequality $n < -12$ on a number line.

c) Write an inequality for the values shown on the number line. Describe a real-life scenario that the inequality might represent.



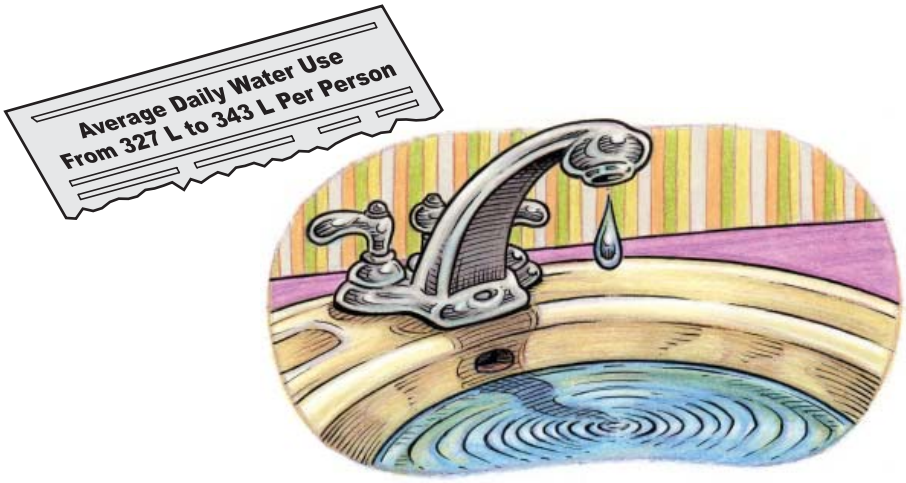
d) Show the possible values for x on a number line, if $-7 \geq x$. What is a different way to express $-7 \geq x$ algebraically?

Example 3: Represent a Combination of Inequalities

Many real life situations can be described by a combination of two inequalities. Represent the situation described in the newspaper headline using inequalities. Show it verbally, graphically, and algebraically.

Did You Know?

Roughly 30% of the water usage in Canadian homes is for flushing the toilet.

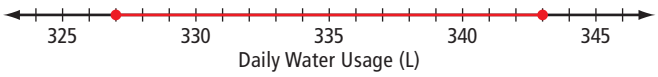


Solution

The newspaper headline describes two inequalities.

Verbally: Daily water use was greater than or equal to 327 L and daily water use was less than or equal to 343 L.

Graphically: Draw a closed circle at 327 and a closed circle at 343. Draw a line segment joining the two circles. This graph represents values that are greater than or equal to 327, and less than or equal to 343.



The values represented by the situation are between and including the boundary points.

Algebraically: Use w to represent the number of litres of water used. You can represent this situation with two inequalities.
$w \geq 327$ and $w \leq 343$

The values that satisfy both inequalities represent the situation.

Show You Know

The most extreme change in temperature in Canada took place in January 1962 in Pincher Creek, Alberta. A warm, dry wind, known as a chinook, raised the temperature from −19 °C to 22 °C in one hour. Represent the temperature during this hour using inequalities. Express the inequalities verbally, graphically, and algebraically.

Key Ideas

- A linear inequality compares linear expressions that may not be equal.
  $x \geq -3$ means that x is greater than or equal to -3.
- Situations involving inequalities can be represented verbally, graphically, and algebraically.

A person must be under twelve years of age to qualify for a child's ticket at the movies. Let a represent the age of the person.

  - Verbally: Use words.

The values of a are less than 12.

  - Graphically: Use visuals, such as diagrams and graphs.

  - Algebraically: Use mathematical symbols, such as numbers, variables, operation signs, and the symbols $<$, $>$, $\leq$, and $\geq$.

The inequality is $a < 12$.

- An inequality with the variable on the right can be interpreted two ways.
  $8 < x$ can be read "8 is less than x." This is the same as saying "x is greater than 8."

Check Your Understanding

Communicate the Ideas

1. Consider the inequalities $x > 10$ and $x \geq 10$.

a) List three possible values for x that satisfy both inequalities. Explain how you know.

b) Identify a number that is a possible value for x in one but not both inequalities.

c) How are the possible values for inequalities involving $>$ or $<$ different than for inequalities involving $\geq$ or $\leq$? Give an example to support your answer.

2. On a number line, why do you think an open circle is used for the symbols $<$ and $>$, and a closed circle for the symbols $\leq$ and $\geq$?

3. Tiffany and Charles have each written an inequality to represent numbers that are not more than 15. Their teacher says that both are correct. Explain why.

Charles: "$15 \geq x$"

Tiffany: "$x \leq 15$"

4. Consider the inequality $x \neq 5$.

a) List at least three possible values for x.

b) How many values are not possible for x? Explain.

c) Explain how you would represent the inequality on a number line.

Practise

For help with #5 to #9, refer to Example 1 on pages 342–343.

5. Write the inequality sign that best matches each term. Use an example to help explain your choice for each.

a) at least

b) fewer than

c) maximum

d) must exceed

6. For which inequalities is 4 a possible value of x? Support your answer using two different representations.

a) $x > 3$

b) $x < 4$

c) $x > -9$

d) $x \geq 4$

7. Write a word statement to express the meaning of each inequality. Give three possible values of y.

a) $y \geq 8$

b) $y < -12$

c) $y \leq 6.4$

d) $y > -12.7$

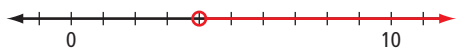
8. At the spring ice fishing derby, only fish 32 cm or longer qualify for the prize categories.

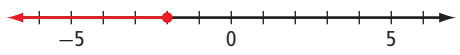
a) Draw a number line to represent the situation.

b) Write a statement to represent the sizes of fish that qualify for prizes.

For help with #9 to #12, refer to Example 2 on pages 343–344.

9. Write a word statement to express each inequality.

a)


b)


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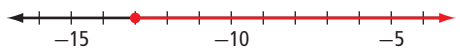
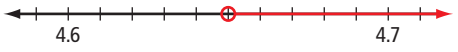


10. Express each inequality algebraically in two different ways.

a)


b)


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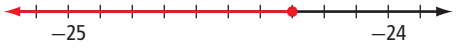


11. Sketch a number line to show each inequality.

a) $x > 3$ **b)** $x < 12$

c) $x \geq -19$ **d)** $-3 \geq x$

12. Represent each inequality graphically.

a) $y \leq 10.7$ **b)** $y \geq -5.3$

c) $y < -\frac{4}{5}$ **d)** $4.8 > x$

For help with #13 to #15, refer to Example 3 on page 345.

13. For each combination of inequalities, show the possible values for x on a number line.

a) $x > 12$ and $x < 17$

b) $x \geq -5$ and $x \leq 0$

c) $x \geq 1\frac{3}{4}$ and $x \leq 4$

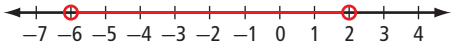
d) $x < -4\frac{1}{2}$ and $x > -11$

14. **a)** Represent the possible values for y graphically, if $y > -9.3$ and $y < -6.7$.

b) Mark any three values on the number line. For each one, explain whether it is a possible value for y.

15. Represent the values shown in red on each number line by a combination of inequalities.

a)


b)


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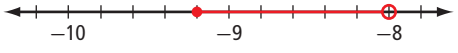


Apply

16. The manager of a clothing store has set goals for her sales staff. Express each goal algebraically.

a) The monthly total sales, m, will be a minimum of \$18 000.

b) At month end, the total time, t, spent counting store inventory will be at most 8 h.

c) The value of total daily sales, d, will be more than \$700.

17. If Emily keeps a daily balance of at least \$1500 in her bank account, she will pay no monthly fees.

a) Draw a number line to represent the situation.

b) If x represents her daily balance, write an inequality that represents the possible values for x when she will pay no fees.

18. Paul is training for a race and hopes to beat the record time. The number line represents the finishing times that will allow him to beat the record.

a) Write a statement to express the finishing times that will let Paul beat the record.

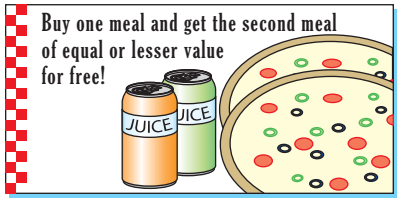
b) Express the inequality algebraically.

19. a) Develop a problem that could be represented by an inequality. Express the inequality verbally.

b) Graph the inequality.

c) Express the inequality algebraically.

20. Owen has a coupon for a restaurant.



a) Owen buys a meal for \$10.75. If m is the cost of his second meal, write an inequality to represent the possible values of m that will allow him to use the coupon.

b) Represent the inequality graphically.

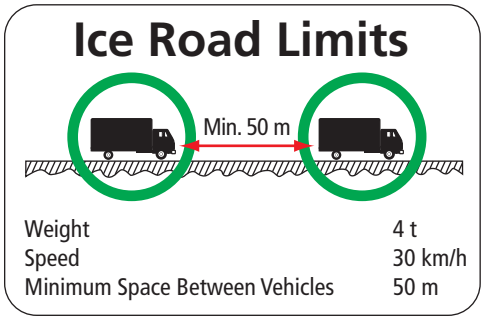
21. Shanelle is buying insurance for a car to drive to and from work. The cost of insurance will be higher if she works farther than 15 km from home.

a) Verbally express the inequality that represents the possible values for the distance for which Shanelle will have to pay more insurance.

b) Sketch a number line to represent the inequality.

22. During winter, ice roads allow access to remote places in northern communities. The ice road to Aklavik, NWT is made through the Mackenzie River Delta. The ice road to Tuktoyuktuk travels up the Mackenzie River and out onto the sea ice. Ice roads are made by flooding the existing ice on a river or lake until it reaches the required thickness.

For safety reasons, there are restrictions such as the ones shown.



Represent each restriction

a) graphically

b) algebraically

Literacy Link

A metric tonne (t) is a measurement of mass that equals 1000 kg.

Extend

23. **a)** If the inequalities $x \geq 6$ and $x \leq 6$ are both true, describe the possible values for x.

b) What would a number line showing possible values of x look like for this situation? Justify your answer.

24. Bluesky is building a wooden puzzle triangle. She has cut two sides that measure 30 cm and 80 cm, respectively. The longest side of the triangle is 80 cm. Write inequalities to represent the possible lengths for the third side of the triangle.

25. What values of x would each of the following combinations of inequalities represent? Explain verbally and show graphically.

a) $x > 4$ and $x < 7$ **b)** $x < 4$ and $x < 7$

c) $x > 4$ and $x > 7$ **d)** $x < 4$ and $x > 7$

Math Link

For safety reasons, some amusement park rides have age and height restrictions for riders.

a) Choose an amusement park ride that you have seen or design one of your own. Describe your ride.

b) For your ride, consider the safety restrictions or conditions that you might impose on riders. List at least three restrictions. Use terms of your choice.

c) Represent each restriction algebraically using a different variable for each.

d) Sketch a sign. Use words and graphics that clearly inform riders about each of your restrictions.