2.3 Proportional Reasoning

Focus on...

After this lesson, you will be able to...

- ❑ solve problems using proportional reasoning
- ❑ use more than one method to solve proportional reasoning problems

When you go snowboarding or skiing, you use proportional reasoning to determine the correct length of ski or board for you to use. This involves using the ratio of your height to the length of the ski or board. To determine the correct width of the board so that your feet do not hang over, you use the ratio of the waist of the board, which is the narrowest part, to your boot size. Riders with small feet need narrower boards than riders with big feet.

When you draw a portrait in art class, you use proportional reasoning to figure out how large to make each facial feature in relation to the other features and how to align the features on the head.

Think about where you have made comparisons. Where might you have used proportional reasoning?

Explore the Math

Materials

- computer access
- short story
- stopwatch
- sticky notes

How do you use proportional reasoning?

1. Copy the following table into your notebook. Put two rows under the column headings to record data for two typists.

Student	Number of Words	Time (min)
		4
		4

2. Work with a partner. Select a short story to type.

Tech Link

You can use a word processing program to count the words for you.

Did You Know?

Typing performance involves both speed and accuracy. Many computer programs provide information about words per minute, number of keystrokes, and error rates.

3. Use a stopwatch and take turns to time each other's typing. The timer tells the typist when to begin and when to stop (after 4 min).

4. **a)** Type at a comfortable rate so you can avoid making many errors.
   b) After the time is up, mark your stopping place in the text using a sticky note.
   c) Count and record the total number of words typed in 4 min.

5. Trade roles and repeat step 4 to get data for your partner.

6. **a)** What is the four-minute typing rate for each typist?
   b) What is the unit rate for each typist?

Reflect on Your Findings

7. **a)** If each typist continued typing at the same rate, how many words could each person type in 1 h? Approximately how many pages is that?
   b) What other factors might affect how long it takes to type the entire story? Give an estimate of the time needed for each typist to type the story.
   c) How did you use a proportion to find your answer to part a)?

proportion

- a relationship that says that two ratios or two rates are equal
- can be written in fraction form:

$$\frac{2}{3} = \frac{6}{9} \quad (\times 3)$$

$$\frac{2\text{ km}}{3\text{ h}} = \frac{6\text{ km}}{9\text{ h}} \quad (\times 3)$$

Example 1: Solve a Rate Problem Using Proportional Reasoning

Electricity costs 11.58¢ for 2 kWh. How much does 30 kWh cost? Give your answer to the nearest cent.

Solution

Method 1: Use a Unit Rate

The cost of 11.58¢ for 2 kWh can be expressed as the rate $\frac{11.58¢}{2\text{ kWh}}$.

Determine the unit rate.

$$\frac{1.58¢}{2\text{ kWh}} = \frac{5.79¢}{1\text{ kWh}}$$

Electricity costs 5.79¢ per kWh or 5.79¢/kWh.

30 kWh costs $30 \times 5.79¢ = 173.7¢$

So, 30 kWh costs 174¢ or $1.74 rounded to the nearest cent.

Wh
divic
by? What number do you divide the numerator by? Are both numbers the same? Why?

173.7 means 173 and seven tenths cents. Recall that five tenths or more is rounded to the next cent.

Method 2: Use a Proportion

Make a proportion to show what you want to find.

$$\frac{11.58¢}{2\text{ kWh}} = \frac{\blacksquare}{30\text{ kWh}}$$

Solve the proportion

$$\frac{11.58¢}{2\text{ kWh}} = \frac{\blacksquare}{30\text{ kWh}}$$

What number do you multiply the denominator by? What number do you multiply the numerator by?

$11.58¢ \times 15 = 173.7¢$

C 11.58 × 15 = 173.7

So, 30 kWh costs about 174¢ or $1.74.

M E

11.58¢ is about 12¢.

$12 \times 15 = (10 \times 15 + 2 \times 15)$

$= 150 + 30$

$= 180$

The answer will be about 180¢ or $1.80.

Show You Know

There are 72 players on 8 baseball teams. Determine the number of players on 2 teams. Show how to find the answer more than one way.

Example 2: Solve a Ratio Problem Using Proportional Reasoning

A wildlife biologist wants to know how many trout are in a slough in Saskatchewan. He captures and tags 24 trout and releases them back into the slough. Two weeks later he returns and captures 30 trout and finds that 5 of them are tagged. He uses the following ratios to estimate the number of fish in the slough:

$$\frac{\text{fish recaptured with tags}}{\text{total fish recaptured}} = \frac{\text{fish caught and tagged}}{\text{total fish in slough}}$$

How many trout does he estimate are in the slough?

Literacy Link

In Western Canada, a *slough* is a small lake or pond formed by rain or melted snow.

Did You Know?

Wildlife biologists can show that these ratios are equal if the fish population has an opportunity to mix before the recapture.

Strategies

Use a Variable

Solution

Method 1: Use a Proportion in Lowest Terms

$$\frac{\text{fish recaptured with tags}}{\text{total fish recaptured}} = \frac{\text{fish tagged}}{\text{total fish in slough}}$$

$\frac{5}{30} = \frac{24}{t}$ Set up the proportion using equal ratios.

$\frac{1}{6} = \frac{24}{t}$ Reduce $\frac{5}{30}$ to $\frac{1}{6}$.

$\times 24$

$\frac{1}{6}$ $\frac{24}{t}$ Make equivalent ratios.

$\times 24$

$t = 6 \times 24 = 144$

The biologist estimates there are 144 trout in the slough.

Method 2: Use the Original Proportion

How would you find the solution if you did not write $\frac{5}{30}$ in lowest terms?

$\frac{5}{30} = \frac{24}{t}$

$\times 4.8$

Divide to find what number multiplied by 5 gives 24.

$\frac{24}{5} = 4.8$

$\frac{5}{30} \quad \frac{24}{t}$

Make equivalent ratios.

$\times 4.8$

$t = 30 \times 4.8$

$t = 144$

The biologist estimates there are 144 trout in the slough.

Show You Know

How much will a dozen erasers cost if three erasers cost 75¢? Show how to find the answer in more than one way.

Key Ideas

- A proportion is a relationship that says that two ratios or two rates are equal.
  - A proportion can be expressed in fraction form:

$\times 5$: $\frac{1}{2} = \frac{5}{10}$: $\times 5$

$\div 3$: $\frac{60 \text{ sit-ups}}{3 \text{ min}} = \frac{20 \text{ sit-ups}}{1 \text{ min}}$: $\div 3$

- You can solve proportional reasoning problems using several different methods.

  A potato farmer can plant three potato plants per 0.5 m^2. How many potato plants can she plant in an area of 85 m^2?

  - Use a unit rate.

$\frac{3 \text{ plants}}{0.5 \text{ m}^2} = \frac{6 \text{ plants}}{1 \text{ m}^2}$ The unit rate is 6 plants/m^2.

$6 \times 85 = 510$

The farmer can plant 510 potato plants.

  - Use a proportion.

$\times 170$: $\frac{3 \text{ plants}}{0.5 \text{ m}^2} = \frac{\blacksquare}{85 \text{ m}^2}$: $\times 170$

Missing value is $3 \times 170 = 510$

The farmer can plant 510 potato plants.

Communicate the Ideas

1. Explain the similarities and differences between a ratio, a rate, and a proportion. Give an example of each one.

2. Your friend missed the lesson on proportions. Explain how to use a proportion to solve this problem.

Cheryl is selling marbles. What is the cost of seven marbles?

Marbles	2	3	4
Cost (¢)	70	105	140

3. a) Write a proportion based on the following scenario:

Three balls cost $1.25. What is the cost of 12 balls?

b) Solve the proportion.

Check Your Understanding

Practise

For help with #4 to #9, refer to Example 1 on pages 64–65.

4. Determine the unit rate.

a) Three dinner rolls cost 99¢.

b) Seven identical objects have a mass of 14 kg.

5. What is the unit rate in each?

a) Two pens cost 94¢.

b) Four blocks stacked one on top of the other are 24 cm high.

6. Delia was paid $35 for 5 h of babysitting. How much should she receive for 3 h? Use a unit rate to find the answer.

7. Solve #6 using a proportion. Show how to find the answer more than one way.

8. Determine the missing value.

a) $\frac{2}{3} = \frac{\blacksquare}{15}$ **b)** $\frac{\blacksquare}{5} = \frac{14}{35}$

c) $\frac{30}{45} = \frac{6}{\blacksquare}$ **d)** $\frac{3}{\blacksquare} = \frac{12}{36}$

9. Determine the missing value to make each rate equivalent. Include the units.

a) $\frac{60 \text{ km}}{3 \text{ h}} = \frac{\blacksquare}{6 \text{ h}}$ **b)** $\frac{\$3}{4 \text{ cans}} = \frac{\$15}{\blacksquare}$

c) $\frac{178 \text{ beats}}{2 \text{ min}} = \frac{\blacksquare}{1 \text{ min}}$ **d)** $\frac{48 \text{ km}}{\$16} = \frac{192 \text{ km}}{\blacksquare}$

For help with #10 to #14, refer to Example 2 on pages 65–66.

10. Set up a proportion for each situation.

a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.

b) There are 13 boys for 15 girls in every classroom at Albany Middle School. If there are 65 boys in the school, then there are 75 girls.

c) On a map, 1 cm represents 25 km. Kendra wants to ride her bike 160 km. This distance is 6.4 cm on the map.

11. A small gear turns 18 times in the same time that a large gear turns 4 times. How many times will the large gear turn if the small gear turns 54 times? Draw a diagram to help set up a proportion and solve the problem.

12. Set up a proportion for each situation using a variable. Do not find the answer.

a) Walter makes his own oil and vinegar dressing. His recipe calls for 175 mL of olive oil and 50 mL of vinegar. What amount of vinegar does he need to mix with 300 mL of olive oil?

b) A baseball player has a home run to strikeouts ratio of 3:17. How many home runs should he hit if he strikes out 187 times?

13. Two quarters have the same value as ten nickels. What is the value of five quarters in nickels?

14. Last night 30 cm of snow fell in 6 h. If it continues snowing at the same rate, how long will it take for 45 cm of snow to fall? Determine the answer two different ways.

Apply

15. Look at the pattern. Set up a proportion you could use to find the number of small squares in Figure 7.

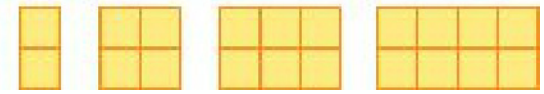


Figure 1 Figure 2 Figure 3 Figure 4

16. A gardener takes a half hour to mow and weed a lawn that measures 20 m by 15 m. He charges $25 per hour. How much should the gardener receive for a lawn that measures 40 m by 30 m?

17. Fresh pickerel is advertised in a local market.

1 kg = 1000 g

a) How much will 6 kg of pickerel cost?

b) Use a proportion to find the cost of 1600 g of pickerel.

18. At an amusement park, a new thrill ride was introduced. It costs $7.50 for 3 rides on the Wild Slider.

a) What is the Wild Slider's unit rate per ride?

b) At this rate, what would it cost for 18 rides on Wild Slider? Determine the answer two different ways.

19. Determine the missing value in each equivalent fraction.

a) $\frac{3}{\blacksquare} = \frac{18}{24} = \frac{\blacksquare}{12}$

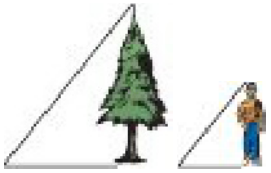
b) $\frac{48 \text{ km}}{\$16} = \frac{144 \text{ km}}{\blacksquare} = \frac{\blacksquare}{\$64}$

20. A breakfast cereal contains corn, wheat, and rice in the ratio of 3 to 4 to 2. If a box of cereal contains 225 g of corn, how much rice does it contain?

21. David can saw a log into three pieces in 7 min. If he continues sawing at a constant rate, how long will it take him to saw a similar log into six pieces?

22. The height of an object compared to the length of its shadow is constant for all objects at any given time.

$$\frac{\text{tree height}}{\text{length of shadow}} = \frac{\text{student height}}{\text{length of shadow}}$$



shadow shadow

Use this information to help answer the following questions.

a) If a 15-m tree casts a 9-m shadow, what is the height of a student who casts a 1.08-m shadow?

b) If a 50-m tower has a shadow 16 m long, how long is the shadow of a student who is 1.5 m tall? Give your answer to the nearest centimetre.

23. According to the *Guinness Book of World Records*, the world's smallest horse is Thumbelina. Thumbelina is 42.5 cm tall and eats about 0.3 kg of food per day. A former world record holder ate food in the same proportion to its height. If it was 46.25 cm tall, how much did it eat? Give your answer to the nearest hundredth of a kilogram.

24. **a)** Describe a pattern you could use to find the next fraction in the following set of fractions. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$

b) Describe a pattern you could use to find the next fraction in the following set of fractions. $\frac{5}{6}, \frac{10}{12}, \frac{15}{18}$

c) Choose any pair of fractions from part a) or part b). Multiply the numerator of one fraction by the denominator of the other fraction. Repeat for two other fractions in the same set. What do you notice about the two products?

d) What prediction could you make about the cross-products of any pair of equivalent fractions? Test your prediction on another pair of equivalent fractions.

Extend

25. Mark estimates that frogs eat six insects per hour and that dragonflies eat nine insects per hour. Assume a frog rests for 8 h each day and a dragonfly rests for 13 h each day. Neither eats while resting.

a) Determine the daily rate of insects eaten by a frog and a dragonfly. Which one eats more insects per day?

b) How many insects would a dragonfly eat in a week?

c) How many insects would a frog eat in August?

26. Two circles have radii with a ratio of 1 to 2. Use a diagram to help answer the following questions.

a) What is the ratio of their circumferences?

b) What is the ratio of their areas?

27. If $a:b = 4:5$, find the ratio of $5a:7b$.

28. The dosage of a certain medicine for a child is 2.5 mL for each 3 kg mass of the child. What is the dose, in millilitres, for a child with a mass of 16.5 kg?

MATH LINK

A horiatiki Greek salad has tomatoes, cucumbers, feta cheese, and olives. It does not contain any lettuce. Many cultures have similar salads. For example, ezme salatasi is a Turkish tomato-cucumber salad with red peppers and paprika, but without the feta and olives.

a) It costs $7.60 to make the horiatiki salad for 12 people. What is the unit price?

b) Choose and write down a recipe for a soup, a salad, or an appetizer that serves between 4 and 6 people. Record how much of each ingredient you will need to serve 10 people at your international meal.