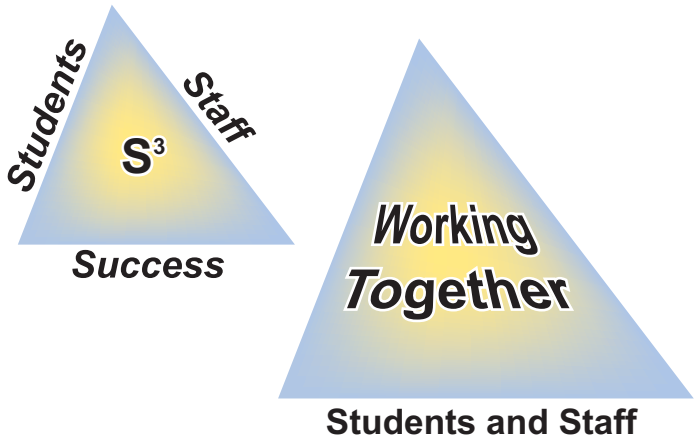
4.3 Similar Triangles

Focus on...

After this lesson, you will be able to...

- determine similar triangles
- determine if diagrams are proportional
- solve problems using the properties of similar triangles



Bonnie and Justin created these logos for the Student Council.

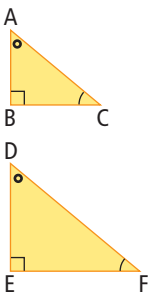
Their advisor tells them that the triangles are similar. How can she tell? What do you know about similar figures? What strategies can you develop to determine if triangles are similar?

Materials

- tracing paper
- ruler
- protractor

corresponding angles
corresponding sides

- have the same relative position in geometric figures



corresponding angles:
∠A and ∠D
∠B and ∠E
∠C and ∠F

corresponding sides:
AB and DE
BC and EF
AC and DF

Explore How to Identify Similar Triangles

1. Trace each logo on separate pieces of tracing paper.

2. a) Measure the angles in each logo. What do you notice about the **corresponding angles**?

b) Measure the side lengths in each logo. What do you notice about the ratios of the **corresponding sides** of the triangles?

Reflect and Check

3. a) What conclusions can you make about the corresponding angles of the two triangles?

b) What conclusions can you make about the corresponding sides of the two triangles?

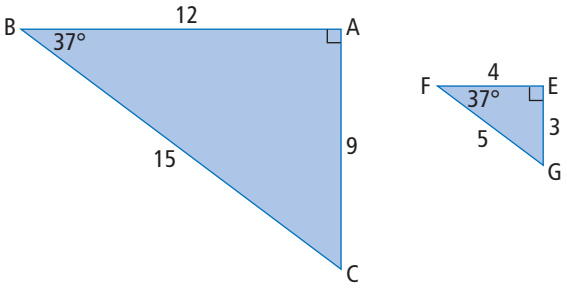
4. a) What conditions do you think are necessary in order for two triangles to be similar?

b) Test the conditions on a different set of two triangles. Are the triangles similar? Discuss with a classmate why you think the triangles are, or are not, similar.

Link the Ideas

Example 1: Identify Similar Triangles

Determine if $\triangle ABC$ is **similar** to $\triangle EFG$.



similar (figures)

- have the same shape but different size
- have equal corresponding angles and proportional corresponding sides

Solution

Similar triangles have corresponding angles that are equal in measure and corresponding sides that are proportional in length.

Compare corresponding angles:
$\angle A = 90°$ and $\angle E = 90°$
$\angle B = 37°$ and $\angle F = 37°$
$\angle C = 53°$ and $\angle G = 53°$

The sum of the angles in a triangle is 180°. If you know the measures of two pairs of angles are equal, then what can you conclude about the third pair of angles?

The corresponding angles are equal.

Compare corresponding sides:

$$\frac{AB}{EF} = \frac{12}{4} = 3 \qquad \frac{BC}{FG} = \frac{15}{5} = 3 \qquad \frac{AC}{EG} = \frac{9}{3} = 3$$

The corresponding sides are proportional with a scale factor of 3.
$\triangle ABC \sim \triangle EFG$

Literacy Link

The symbol $\sim$ means is similar to.

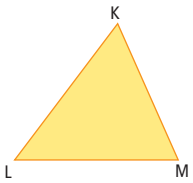
$\triangle ABC \sim \triangle EFG$ means triangle ABC is similar to triangle EFG.

Literacy Link

Angles can be named in two ways:

- Use three capital letters. The middle letter is the vertex of the angle.
- Use only the middle letter identifying the vertex. Use a single letter when there is only one angle at a vertex.

For example, the angle at vertex L can be named $\angle KLM$ or $\angle L$.



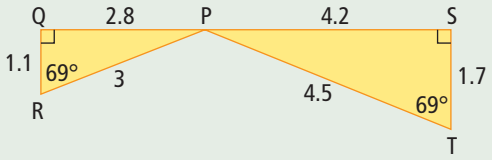
WWW Web Link

To learn more about properties of similar triangles, go to www.mathlinks9.ca and follow the links.

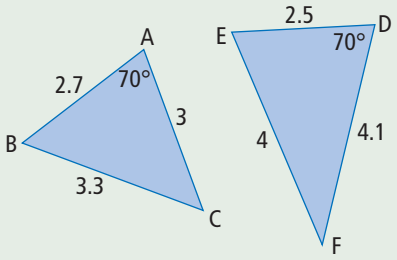
Show You Know

Determine if each pair of triangles is similar. Show how you know.

a)



b)

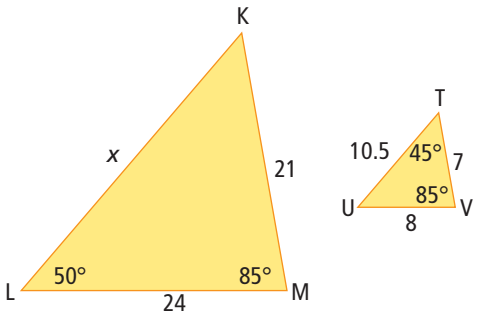


Example 2: Use Similar Triangles to Determine a Missing Side Length

Kyle is drawing triangles for a math puzzle. Use your knowledge of similar triangles to determine

a) if the triangles are similar

b) the missing side length



Solution

Strategies
Organize, Analyse, and Solve

a) Check that $\triangle KLM$ is similar to $\triangle TUV$.

The sum of the angles in a triangle is 180°.

$\angle K = 180° - 50° - 85°$
$= 45°$

$\angle U = 180° - 85° - 45°$
$= 50°$

Compare corresponding angles:

$\angle K = 45°$ and $\angle T = 45°$

$\angle L = 50°$ and $\angle U = 50°$

$\angle M = 85°$ and $\angle V = 85°$

All pairs of corresponding angles are equal.

Therefore, $\triangle KLM \sim \triangle TUV$.

It is not necessary to prove both conditions for similarity. One is sufficient.

b) You can compare corresponding sides to determine the scale factor.

$\frac{LM}{UV} = \frac{24}{8}$ $\quad$ $\frac{KM}{TV} = \frac{21}{7}$ $\quad$ $\frac{KL}{TU} = \frac{x}{10.5}$

$= 3$ $\quad$ $= 3$ $\quad$ $=$ ■

The scale factor is 3. You can solve for the unknown length.

Method 1: Use a Scale Factor

Since the triangles are similar, you can use the scale factor to determine the missing length.

$$\frac{x}{10.5} = 3$$

$$x = 31.5$$

How would you solve for x?

The missing side length is 31.5 units.

Method 2: Use a Proportion

Since the triangles are similar, you can use equal ratios to set up a proportion.

$$\frac{KM}{TV} = \frac{KL}{TU}$$

$$\frac{21}{7} = \frac{x}{10.5}$$

$\times 1.5$ $\times 1.5$

$10.5 \div 7 = 1.5$

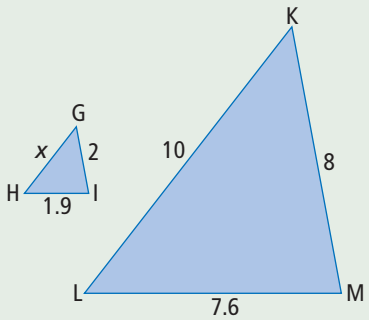
$$x = 31.5$$

The missing side length is 31.5 units.

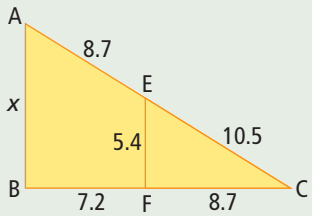
Show You Know

Solve using a method of your choice.

a) $\triangle GHI$ is similar to $\triangle KLM$. What is the missing side length? Express your answer to the nearest tenth.



b) $\triangle ABC$ is similar to $\triangle EFC$. Determine the missing side length. Express your answer to the nearest tenth.



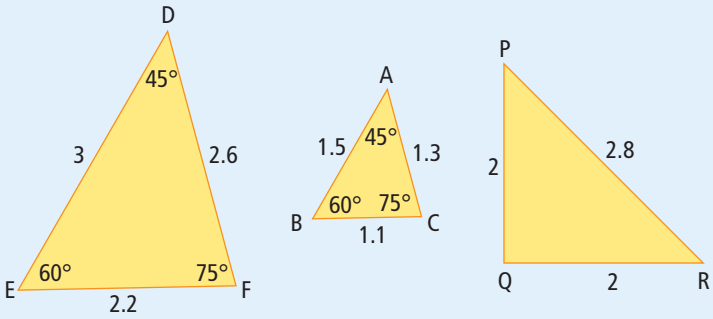
Key Ideas

- Triangles are similar if one of the following conditions holds true:
  - corresponding angles are equal in measure
  - corresponding sides are proportional in length

  $\triangle DEF$ is similar to $\triangle ABC$.
  $\triangle DEF$ is not similar to $\triangle PQR$.
  $\angle D = \angle A, \angle E = \angle B, \angle F = \angle C$

  $\frac{DE}{AB} = \frac{3}{1.5} = 2 \qquad \frac{EF}{BC} = \frac{2.2}{1.1} = 2 \qquad \frac{DF}{AC} = \frac{2.6}{1.3} = 2$

- You can solve problems related to similar triangles using different methods.
  - Use a scale factor.
  - Use a proportion.

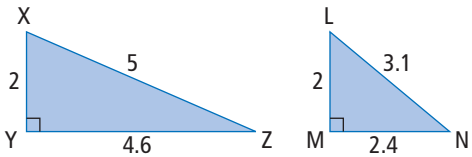


Check Your Understanding

Communicate the Ideas

1. If two triangles are similar, what can you say about the angles of the triangles? the side lengths of the triangles?

2. Amanda is unclear about similar triangles. She drew these two triangles and states they are similar. Is she correct? Explain.

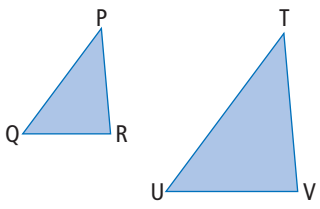


3. Are two triangles that have equal angles and equal sides similar? Use an example to support your answer.

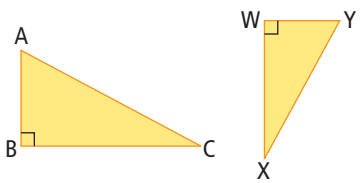
Practise

For help with #4 to #8, refer to Example 1 on page 147.

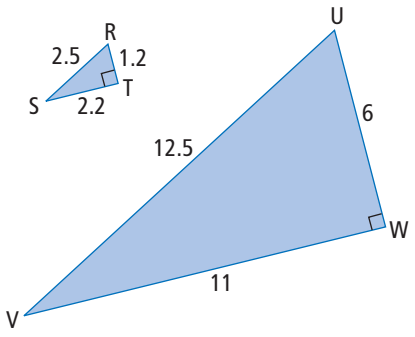
4. List the corresponding angles and the corresponding sides for $\triangle PQR$ and $\triangle TUV$.



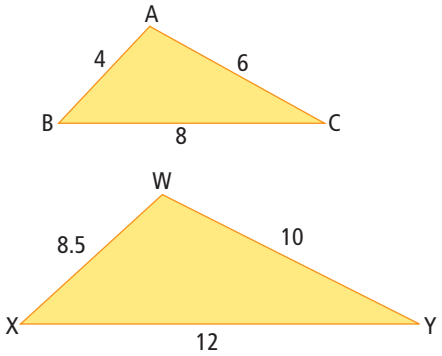
5. What are the corresponding angles and the corresponding sides in this pair of triangles?



6. Are the triangles similar? Show how you know.



7. Determine if the triangles are similar. Show how you know.

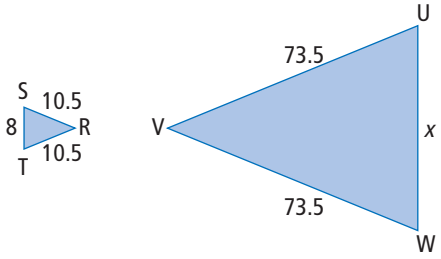


8. Determine which pairs of triangles are similar. Use a sketch to help explain how you know.

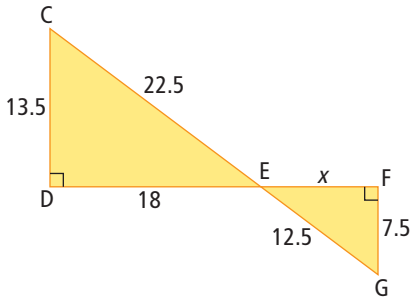
Triangle	Angles	Sides
△ABC	∠A = 90° ∠B = 45° ∠C = 45°	AB = 6 BC = 8.4 AC = 6
△EFG	∠E = 90° ∠F = 45° ∠G = 45°	EF = 3 FG = 4.2 EG = 3
△HIJ	∠H = 90° ∠I = 60° ∠J = 30°	HI = 9.2 IJ = 18.4 HJ = 15.9
△KLM	∠K = 90° ∠L = 45° ∠M = 45°	KL = 9 LM = 12.6 KM = 9

For help with #9 to #11, refer to Example 2 on pages 148–149.

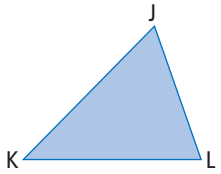
9. △STR is similar to △UWV. Determine the missing side length.



10. △CDE is similar to △GFE. What is the missing side length?

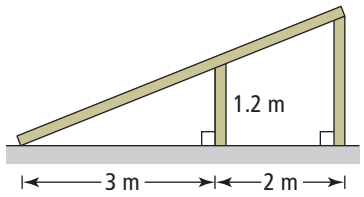


11. Draw a triangle that is similar to the one shown. Label the measurements for angles and sides on your similar triangle.

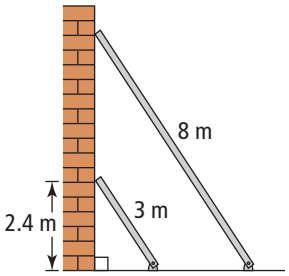


Apply

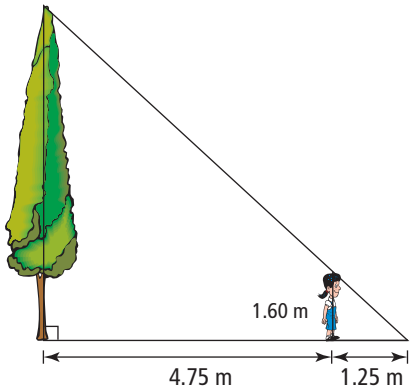
12. Sam built a ramp to a loading dock. The ramp has a vertical support 2 m from the base of the loading dock and 3 m from the base of the ramp. If the vertical support is 1.2 m in height, what is the height of the loading dock?



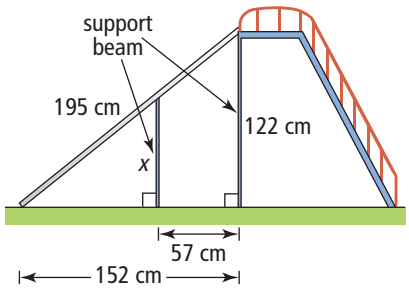
13. Two extension ladders are leaning at the same angle against a vertical wall. The 3-m ladder reaches 2.4 m up the wall. How much farther up the wall does the 8-m ladder reach?



14. Erin, who is 1.60 m tall, casts a shadow that is 1.25 m long. Her shadow extends to the end of a tree's shadow when she stands 4.75 m from the tree. What is the height of the tree?



15. Sara was helping her father assemble a slide for the local park. He decides to reinforce the slide with an extra support beam. How long should the extra support beam be?

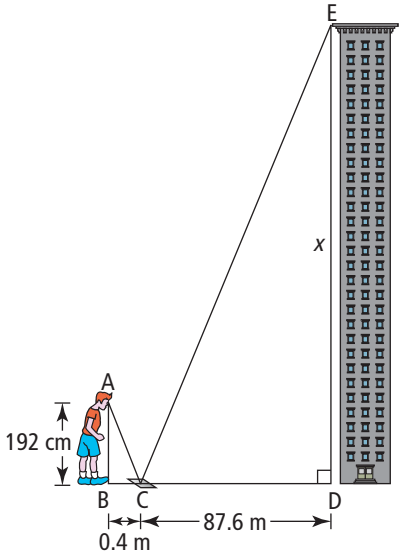


16. Peter, who is 168 cm tall, casts a 45-cm shadow. Michael, who is standing beside him, casts a 40-cm shadow. Can you tell who is taller? Use a diagram to help explain why or why not.

17. Develop a word problem that can be solved using similar triangles. Include a diagram.

Extend

18. A tourist wants to estimate the height of an office tower. He places a mirror on the ground and moves away to sight the top of the tower in the mirror.



a) How tall is the tower?

b) In this situation, why is the mirror reflection a better way to indirectly measure the tower than by using shadows?

19. Is it possible for the two triangles described below to be similar? Explain your reasoning.

a) Two angles of one triangle measure 60° and 70°. Two angles of the other triangle measure 50° and 80°.

b) Two angles of one triangle measure 45° and 75°. Two angles of the other triangle measure 45° and 60°.

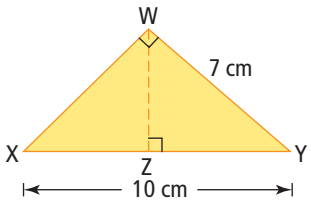
20. The sides of a triangle measure 3 cm, 5 cm, and 6 cm. If the side of a similar triangle corresponding to 3 cm measures 8 cm,

a) determine the lengths of the other sides

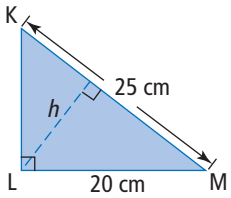
b) determine the ratio of the perimeter of the smaller triangle to the perimeter of the larger triangle

21. Using a measuring tape, your shadow, and yourself, how can you determine the height of your school without actually measuring it?

22. $\triangle WXY$ is similar to $\triangle ZWY$. Calculate ZY to the nearest tenth.

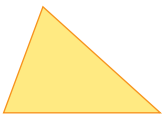


23. Use two different sets of measurements to determine the area of $\triangle KLM$.



Math Link

For your design project report, include a signature logo that features your name.



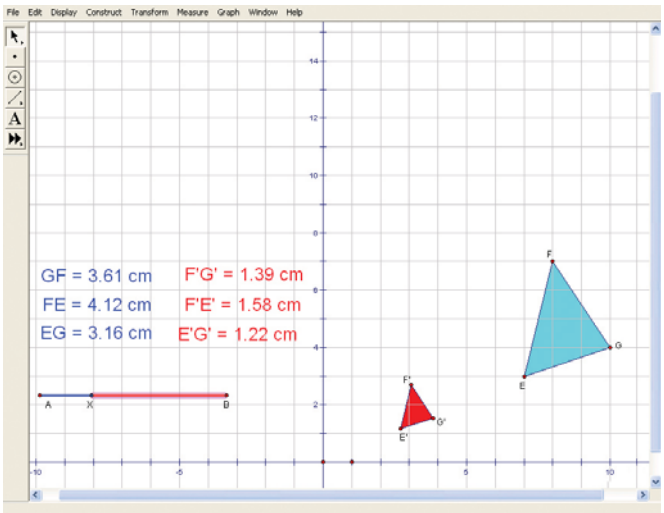
a) On a sheet of 8.5 × 11 paper, design your logo. Include a triangle that is similar to the one shown. Measure all the angles and side lengths.

b) Draw a scale diagram of the logo to fit on your design project. Identify the scale factor you used.

Tech Link

Similarity and Scale Factors

In this activity, you can use dynamic geometry software to explore similarity and scale factors. To use this activity, go to www.mathlinks9.ca and follow the links.



Explore

1. Slide point X along line segment AB and describe what happens to the image drawing.

2. How do the measures of the corresponding sides of the drawing change relative to each other? Explain.

3. Compare the scale factor to the lengths of the sides of the original drawing and the image drawing. Create and complete a table similar to the one below with measurements taken at different locations. Discuss your findings with a classmate.
Hint: In the table, *m* means *the measure of*.

mFE	mF′E′	$\frac{m\text{AX}}{m\text{XB}}$