# Scale Diagrams

Car manufacturers create scale drawings that show what a new car will look like.

An actual car measures 339.2 cm in length and 163.2 cm in height. It is drawn to a scale of 1 : 32. Is the drawing an accurate representation of the actual model? What different strategies can you develop to find out?

## Explore the Accuracy of a Diagram

**1.** What measurements would help you compare the diagram of the car to the actual car? Take the measurements.

**2.** Compare the measurements. What conclusions can you make?

### Reflect and Check

**3. a)** How did you set up your calculations to determine if the diagram accurately represents the actual car?

**b)** What information did you need to determine whether the diagram is an accurate representation of the actual car?

**4. a)** Choose an object and draw one view of it. Estimate the scale between your drawing and the actual object.

**b)** Use the method you developed to determine how accurately the drawing represents the actual object.

**5.** Compare your method with the one used by a classmate. How are the methods similar? How are they different? Which method seems more efficient? Explain.

# Link the Ideas

## Example 1: Use the Scale to Determine the Actual Length of an Object

The **scale diagram** of a skateboard uses a scale of $1:14$. What is the actual length of the skateboard?



5.5 cm
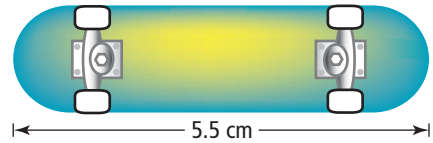
### Solution

**Method 1: Use the Scale**
The scale $1:14$ means that the actual dimensions of the skateboard are 14 times those of the diagram. Multiply the length of the skateboard in the diagram by 14.
$5.5 \times 14 = 77$

The actual length of the skateboard is 77 cm.

**Method 2: Use a Proportion**
Set up a proportion using the scale and the measurement that is given.

$$\text{scale} = \frac{\text{diagram measurement}}{\text{actual measurement}}$$

> The scale is $1:14$. The diagram measures 5.5 cm. The actual measurement is unknown.

$$\frac{1}{14} = \frac{5.5}{y}$$

$\times 5.5$

$$\frac{1}{14} = \frac{5.5}{77}$$

$\times 5.5$

The actual length of the skateboard is 77 cm.

### Show You Know

The scale for the diagram of the chinook salmon is $1:9.2$.



5 cm

Calculate the actual length of the salmon.

# Example 2: Determine the Scale Factor

An actual Canadian quarter has a diameter of 23.88 mm. Calculate the scale factor used to create the diagram of the quarter. Express the answer to the nearest tenth.

### Solution

Measure the diameter of the diagram of the quarter. It measures 1.4 cm.

> The diagram is a reduction. The scale factor will be less than 1.

Set up a proportion for the scale and the measurements.

$$\text{scale} = \frac{\text{diagram measurement}}{\text{actual measurement}}$$
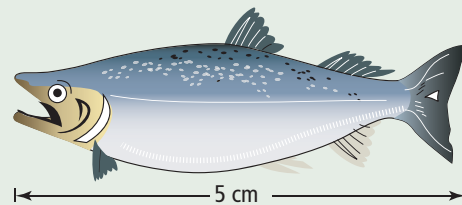
$$\frac{1}{x} = \frac{14}{23.88}$$

$\div 14$

$$\frac{1}{1.7} = \frac{14}{23.88}$$

$\div 14$

> To compare items using a ratio, the units must be the same. The actual measurement is 23.88 mm. The diagram measures 1.4 cm, which is 14 mm.

**Strategies**

**Solve an Equation**

Divide to determine the scale factor.
$1 \div 1.7 \approx 0.588\ldots$
$\qquad \approx 0.6$

The scale factor is approximately 0.6.

This means that the quarter in the diagram is approximately 0.6 times as large as the actual quarter.

## Show You Know

The flying distance from Dawson City to Whitehorse is 540 km. The distance shown on the map is 3 cm.

**a)** Complete the following to express the map scale in words.

scale: 1 cm represents ■ km

**b)** What is the scale factor?
Hint: 1 km = 100 000 cm.

Yukon

Dawson

Whitehorse

## Key Ideas

- A scale diagram is a proportionally smaller or larger representation of an actual object.

A map is a scale diagram.



- The scale is a ratio between two sets of measurements.

  The scale compares a distance on the map to the actual distance.

  If 1 cm represents 12 km, then 1 cm represents 12 × 100 000 cm.

  The scale is 1 : 1 200 000.

  The scale factor is $\dfrac{1}{1\,200\,000}$.
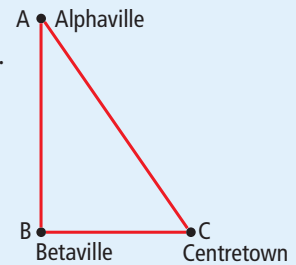
  > 1 km = 1000 m and 1 m = 100 cm, so 1 km = 100 000 cm.

- You can solve problems involving scale diagrams using different methods.

  - Use a scale.

    The distance from A to B on the map is 3 cm. Determine the actual distance.

    3 × 1 200 000 = 3 600 000

    The actual distance is 3 600 000 cm or 36 km.

  - Use a proportion.

    The distance from A to C on the map measures 4 cm. Determine the actual distance.

    $$\text{scale} = \dfrac{\text{diagram measurement}}{\text{actual measurement}}$$

    $$\dfrac{1}{1\,200\,000} = \dfrac{4}{\blacksquare}$$

    The actual distance is 4 800 000 cm or 48 km.

# Check Your Understanding

## Communicate the Ideas

1. Joseph is unsure about how to determine the actual length of an object using a scale diagram. List the steps to solve a problem of your choice. Discuss the steps with a classmate.

2. Kira plans to ride 150 km on her bike. This distance is 10 cm on a map. Express the scale of the map

   a) in words          b) as a ratio

3. How can you check that the larger image of the airliner is proportional to the dimensions in the original photo? Try out your method. Describe your results.



original

image

## Practise

**4.** State whether you would multiply or divide to determine the missing value.

**a)** $\dfrac{1}{3} = \dfrac{\blacksquare}{144}$      **b)** $\dfrac{1}{\blacksquare} = \dfrac{5.2}{117}$

**5.** Determine the missing value in each proportion.

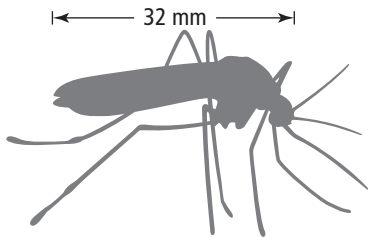**a)** $\dfrac{1}{9} = \dfrac{\blacksquare}{117}$      **b)** $\dfrac{1}{12} = \dfrac{10.5}{\blacksquare}$

**6.** Calculate the actual length of each object.

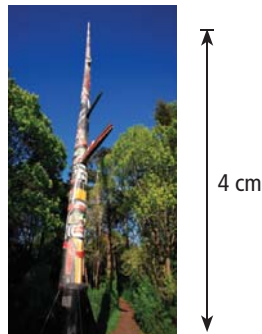**a)** The scale for the image of the school bus is $1:302.5$.

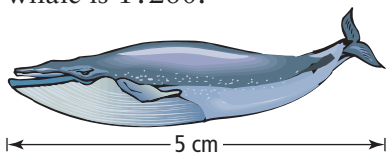**b)** The scale for the enlarged image of a mosquito is $1:0.5$.

$\longleftarrow$ 32 mm $\longrightarrow$

**7.** Determine the actual length of each object.

**a)** The scale for the image of Victoria's tallest totem pole is $1:972.5$.

4 cm

**b)** The scale for the model of the humpback whale is $1:280$.

$\longleftarrow$ 5 cm $\longrightarrow$

**8.** What is the scale factor?

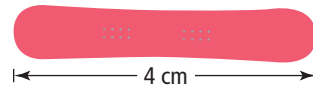**a)** $\blacksquare = \dfrac{30}{200}$      **b)** $\blacksquare = \dfrac{21}{12.5}$

**9.** Determine the scale factor.

**a)** $\blacksquare = \dfrac{0.5}{25}$      **b)** $\blacksquare = \dfrac{1.6}{3.2}$

**10.** What scale factor was used to create the image of the snowboard if its actual length is 166 cm? Express your answer to the nearest hundredth.

$\longleftarrow$ 4 cm $\longrightarrow$

**11.** At the time his photo was taken for the hockey card, Ken was 152.4 cm tall. Calculate the scale factor used to create Ken's image on the hockey card. Express the answer to the nearest hundredth.

6 cm
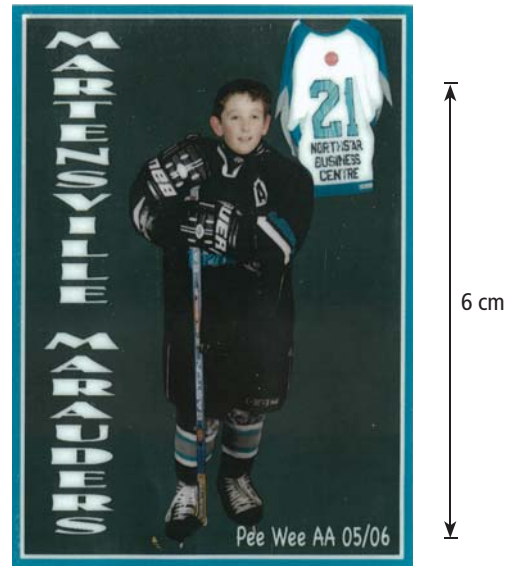
**12.** A flying distance is 800 km. If this distance on a map is 5 cm, what is the scale factor? Hint: 1 km = 100 000 cm.

## Apply

**13.** A Ukrainian decorated egg is called a pysanka. A giant version of a pysanka is located in Vegreville, Alberta. The length of the egg is 9.4 m.



a) On a scale diagram of the pysanka, what would the length be, if you used a scale of 1:150?

b) Could your result represent the length of an actual egg? Explain.

**14.** The footprint of an adult male polar bear measures 30 cm across.

a) What is the scale factor of this drawing?

b) What is the actual length of the polar bear's footprint? Show how you know.

c) Measure your hand span by spreading your hand on a piece of paper. Write the ratio of your hand span to the span of the polar bear's footprint. What conclusion can you make?



**15.** Viruses are much smaller than bacteria. Some viruses measure 0.0001 mm in diameter. An artist's diagram of a virus shows the diameter as 5 mm. Determine the scale factor used.

**16.** For the science fair, Leanne plans to build a scale model of a communications tower that is actually 250 m in height. The model has to fit in the foyer of the school, which has a floor-to-ceiling height of 3 m. If Leanne uses a scale of 1:100 to build the model, will it fit into the foyer? Show your work.

**17.** A model train is a scale model created from actual measurements. The scale factor for HO or Half Zero model trains is 1:87. A typical engine, such as the one shown, is 50 mm in height and 200 mm in length. Determine the actual dimensions of the train engine.



**18.** Determine the scale factor for each enlargement or reduction.

a) from A to B      b) from A to C

c) from B to C      d) from C to A

e) from C to B
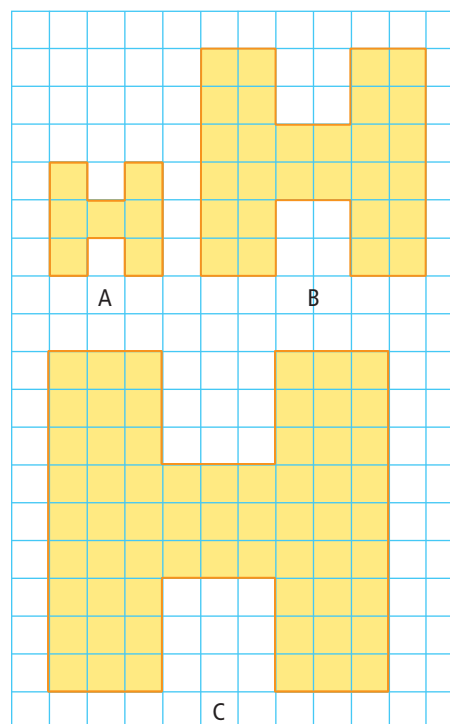
**19.** Tracy took a picture of a wind turbine at the wind farm in Cowley Ridge, Alberta. The height of the turbine is 45 m.



2.5 cm

**a)** What scale factor was used to make this reduction?

**b)** What is the length of a wind turbine blade?

## Extend

**20.** △ABC has coordinates A(4, 3), B(4, 0), and C(7, 0). △DEF has coordinates D(0, −1), E(0, −2), and F(1, −2).

**a)** Draw the triangles on grid paper.

**b)** Are the two triangles proportional to each other? Justify your answer.

**c)** What is the scale factor of △ABC to △DEF?

**d)** Determine the scale factor of △DEF to △ABC.

**e)** Calculate the area of each triangle.

**f)** What is the ratio of the area of △ABC to the area of △DEF? of the area of △DEF to the area of △ABC?

**g)** How does the scale factor of the side lengths compare to the scale factor of the areas?

**21.** Elk Valley Coal uses trucks such as the one shown. The man in the picture is 1.69 m tall.



**a)** What is the height of the wheel of the truck?

**b)** What is the height of the truck?

> **Did You Know?**
>
> Elk Valley Coal operates five open-pit coal mines. The mines are in southeastern British Columbia and in west-central Alberta.

**22.** A rectangle has sides measuring 12 cm and 16 cm. An enlarged, similar rectangle has an area of 1200 cm². 

**a)** What is the scale factor between
  • the smaller and the larger rectangle?
  • the larger and the smaller rectangle?

**b)** Is one method better than the other to express this scale factor? Explain your reasoning.

## Math Link

**a)** Determine the scale factor for the enlargement or reduction of the design you drew for the Math Link on page 138. Show your work.

**b)** Choose a new feature to add to your design.
  • Draw it on your scale diagram.
  • Calculate the actual dimensions of the new feature.

**c)** Explain how you know the scale diagram is proportional to the actual design.